FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
FULL TEST – III
PAPER –1
TEST DATE: 08-02-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. A, D

Sol. For transverse wave, $v_T = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{\rho \times S}}$

$$V_{Al} = \sqrt{\frac{900}{2.5\times10^{3}\times10^{-6}}} = \frac{30}{50\times10^{-3}} = 600 \text{ m/s}$$

$$V_{Cu} = \sqrt{\frac{900}{8.1\times10^{3}\times10^{-6}}} = \frac{30}{90\times10^{3}} = \frac{1000}{3} \text{m/s}$$

$$\frac{a_r}{a_i} = \frac{V_{Al} - V_{Cu}}{V_{Al} + V_{Cu}} = \frac{\frac{800}{3}}{\frac{2800}{3}} = \frac{2}{7} \approx 0.3$$

For longitudinal wave $V_\ell = \sqrt{\frac{Y}{\rho}}$

$$V_{Al} = \sqrt{\frac{9\times10^{10}}{2.5\times10^{3}}} = \frac{3\times10^{5}}{50} = 6000 \text{ m/s}$$

$$V_{Cu} = \sqrt{\frac{16\times10^{10}}{8.1\times10^{3}}} = \frac{4\times10^{5}}{9\times10} = \frac{40000}{9} \text{ m/s}$$

$$\frac{a_r}{a_i} = \frac{V_{Al} - V_{Cu}}{V_{Al} + V_{Cu}} = \frac{\frac{14000}{9}}{\frac{94000}{9}} = \frac{7}{47} \approx 0.15$$

2. A, C, D

Sol. Work done by force equal to the elastic potential energy stored in the wire.

3. A, C

Sol. $P_1 = \frac{K_1 A(100-0)}{L} \Rightarrow Q = P_1 t_1$

$$P_2 = \frac{K_2 A(100-0)}{L} \Rightarrow Q = P_2\, t_2$$

$$P_3 = \frac{P_1P_2}{P_1+P_2} = \frac{\frac{Q}{t_1}\times\frac{Q}{t_2}}{\frac{Q}{t_1}+\frac{Q}{t_2}} = \frac{Q}{t_1+t_2}$$

$$\Rightarrow \quad Q = P_3(t_1+t_2)$$

$$\Rightarrow t_{10} = t_1 + t_2 = 100 \text{ minutes}$$

$$P_2 = P_1 + P_2 = \frac{Q}{t_1}+\frac{Q}{t_2} = \frac{Q(t_1+t_2)}{t_1t_2}$$

$$\Rightarrow \quad Q = P_4\left(\frac{t_1t_2}{t_1+t_2}\right) \Rightarrow t_{20} = \frac{t_1t_2}{t_1+t_2} = \frac{20\times 80}{100} = 16 \text{ min}$$

4. D

Sol. When current is increasing magnetic field is increasing. Hence current flows in opposite fashion by Lenz's law.

5. B

Sol. $I_0 = K\ell^4$

$I_C \Rightarrow$ moment of inertia about an axis passing through 'C' and perpendicular to the plane.

So, $I_C = 2K(\ell\sqrt{2})^4 - 2k\ell^4 + \frac{I_0}{2}$

$= 8k\ell^4 - 2k\ell^4 + \frac{I_0}{2} = 6.5I_0$

6. D

Sol. $Al\sigma g - Ax\,\rho g = Al\sigma a$

$a = g - \frac{\rho g x}{\sigma \ell}$

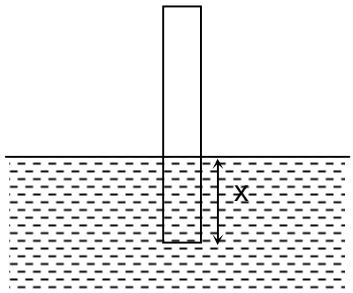
$\int_0^v v.dv = \int_0^x g - \frac{\rho g x}{\sigma\ell}dx$

$\Rightarrow \frac{v^2}{2} = gx - \frac{\rho g}{\sigma\ell}\frac{x^2}{2}.$

At maximum displacement,

$v = 0$

$\Rightarrow x = \frac{2\sigma\ell}{\rho} = 2\,x\,\frac{1}{2}\,x\,4 = 4m$



7. D

Sol. Extra path by $S_2 = (d\sin\theta)\mu_w = (\mu_g - \mu_w)t$

$\frac{2}{3}\times\frac{1}{2}\left(\frac{4}{3}\right) = \left(\frac{1}{6}\right)t$

$t = \frac{8}{3}mm$

8. C

Sol. Velocity of the wave in the wire *AB*

$$v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{160}{0.12/4.8}} = 80 \text{ m/s}$$

Velocity of the wave in the wire BC

$$v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{160}{0.4/2.56}} = 32 \text{ m/s}$$

Amplitude of the reflected wave

$$a_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) a_i = \frac{32-80}{32+80} \times 3.5 = -1.5 \text{ cm}$$

Negative sign signifies that reflected wave is inverted.

Amplitude of the transmitted wave $a_t = \frac{2v_2}{v_1 + v_2} a_t = \frac{2 \times 32}{32+80} \times 3.5 = 2.0 \text{ cm}$

Equation of the reflected wave $y_r = -1.5\sin(\omega t + kx)$

$\therefore$ Resultant wave equation in the wire *AB*

$$y = 3.5\sin(\omega t - kx) - 1.5\sin(\omega t + kx)$$

$$y = 1.5\sin(\omega t - kx) - 1.5\sin(\omega t + kx) + 2.0\sin(\omega t - kx)$$

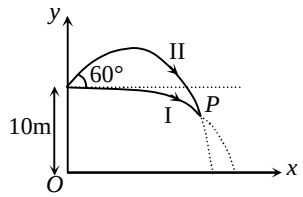
$$y = -3.0\sin kx \cos \omega t + 2.0\sin(\omega t - kx)$$

Maximum displacement of the nodes in the wire *AB* = 2.0 cm

Fraction of power transmitted in the wire $BC = 1 - \frac{a_r^2}{a_i^2} = 1 - \frac{1.5 \times 1.5}{3.5 \times 3.5} = 0.82$

9. A

Sol. As shown in figure, from equation of motion



$s = s_0 + ut + \frac{1}{2}at^2$ for first gun,

$$x_1 = 5\sqrt{3}\, t_1 \quad \text{...(i)}$$

and $y_1 = 10 - \frac{1}{2} g t_1^2$ ...(ii)

While for the second gun

$$x_2 = 5\sqrt{3}\cos 60° \, t_2 \quad \text{...(iii)}$$

and $y_2 = 10 + 5\sqrt{3}\sin 60° t_2 - \frac{1}{2} g t_2^2$ ...(iv)

For collision, $x_1 = x_2$ and $y_1 = y_2$

i.e. $5\sqrt{3}\, t_1 = 5\sqrt{3} \times \frac{1}{2} t_2$ and $10 - \frac{1}{2} g t_1^2 = 10 + \frac{15}{2} t_2 - \frac{1}{2} g t_2^2$

i.e., $t_2 = 2t_1$ and $g(t_2^2 - t_1^2) = 15 t_2$

which on simplification gives $t_1 = 1$ sec and $t_2 = 2$ sec (as $t_2 \neq 0$)

and substituting $t_1 = 1$ s and $g = 10$ m/s^2 in equations (i) and (ii) co-ordinates of point *P* will be $(5\sqrt{3}, 5)$ m.

$v_{y_2} = 5\sqrt{3}\sin 60° - 10 \times 2$ = –12.5 m/s

$v_{y_1} = -10 \times 1 = -10$ m/s

So, magnitude of *y*-component of relative velocity just before collision = 2.5 m/s

10. A

Sol. Conceptual

11. D

Sol. $\phi_1 = \sin^{-1}\dfrac{L}{r} = \sin^{-1}\dfrac{\left(\dfrac{mv_0}{2qB}\right)}{\dfrac{mv_0}{q_B}} = \dfrac{\pi}{6}$

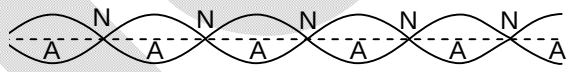
$\phi_2 = \sin^{-1}\dfrac{\sqrt{3}}{2} = \dfrac{\pi}{3}$

Time spent in magnetic field $= \dfrac{\delta m}{qB_0}$, δ= angle of deviation.

SECTION – B

12. 2

Sol. The end which is clamped will be node and the free end will be an antinode. The nodes and antinodes will be formed as shown.



Let λ be the wave length of the constituent waves

Then, length of the rod $= \dfrac{5\lambda}{2} + \dfrac{\lambda}{4} = \dfrac{11\lambda}{4} \Rightarrow \dfrac{11\lambda}{4} = 1 \Rightarrow \lambda = \dfrac{4}{11}$m

Speed of the constituent waves $v = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{6.4 \times 10^{10}}{4 \times 10^3}} = 4000$ m/s

Frequency $f = \dfrac{v}{\lambda} = \dfrac{4000}{4/11} = 11000$Hz.

13. 8

Sol. n = 8

$F = \dfrac{\sigma^2}{2\epsilon_0}\pi(R^2 - h^2)$

14. 9

Sol. $g_1 = g\left(1 - \dfrac{h_1}{R}\right)$ $\quad \therefore \ \Delta g_1 = -\dfrac{g}{R}\Delta h_1$

and $g_2 = g\dfrac{R^2}{(R+h_2)^2}$ $\quad \therefore \ \Delta g_2 = -\dfrac{2gR^2}{(R+h_2)^3}\Delta h_2$

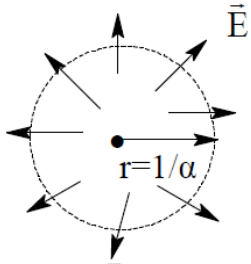
given that, $\Delta g_1 = \Delta g_2$

So, $h_2 = R\left[2^{1/3} - 1\right] = 900$ km

15. 1

Sol. As the electric field is radial, by applying gauss law, we can write

$$\int \vec{E}.\overrightarrow{ds} = \frac{Q}{\varepsilon_0}$$



For $r = \frac{1}{\alpha}, \vec{E} = C\left(1 - e^{-\alpha \times 1/\alpha}\right)\frac{\hat{r}}{(1/\alpha)^2}$

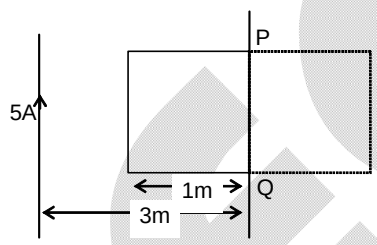
$\therefore \oint \vec{E}.\overrightarrow{ds} = C(1 - e^{-1})\alpha^2 \times 4\pi(1/\alpha)^2$

$\Rightarrow \frac{Q}{\varepsilon_0} = 4\pi C(1 - e^{-1}) \Rightarrow Q = (1 - e^{-1}) \Rightarrow \therefore N = 1$

16. 7

Sol. EMF induced $= -\frac{d\phi}{dt}$

current induced $= \frac{1}{R}\left|\frac{d\phi}{dt}\right| = i$



$\Rightarrow \quad idt = \frac{d\phi}{R}$

Integrating, $\int idt = \int \frac{d\phi}{R}$ = net change in flux /R

Hence charge flown $= \frac{\text{net change in flux}}{R}$

Now net change in flux $= \phi_i - \phi_f = \frac{\mu_0(1)(5)}{2\pi}\left[\ln\left(\frac{3}{2}\right) - \ln\left(\frac{4}{3}\right)\right]$ Weber

charge flown $= 1.17 \times 10^{-7}$ coulomb $\Rightarrow x = 1$

17. 1

Sol. Find μ_{min} and μ_{max} and then locate the images along x-direction.

$\mu_{min} = \frac{3}{2}$ and $\mu_{max} = 3$

Chemistry

PART – II

SECTION – A

18. B, D
Sol. Bond energy $Cl_2 > Br_2 > F_2$
Electron affinity Cl > F > Br

19. D
Sol. Hybridisation doesn't take place in diatomic molecules.

20. C, D
Sol. $[Cr(NH_3)_6]^{3+}$ is paramagnetic
$[CoI_4]^{2-}$ is tetrahedral in geometry

21. B
Sol. Weight of H_2O in solution = $75 \times 0.08 = 6g$

$$(CH_3CO)_2O + H_2O \longrightarrow 2CH_3COOH$$

$$\frac{W}{102} \qquad \frac{6}{18}$$

$$\frac{W}{102} = \frac{6}{18} \Rightarrow W = \frac{6\times102}{18} = 34\,g$$

$\therefore$ weight of acetic anhydride required is 34 g

22. A
Sol.

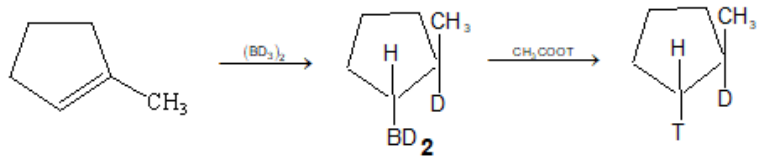
$$(CH_3)_2CH - \underset{(g)}{N} = N - CH \longrightarrow N_2(g) + C_6H_{14}(g)$$

$$P_1 \qquad - \qquad -$$
$$Pi - x \qquad x \qquad x$$

$$P = Pi - x + x + x = Pi + x \Rightarrow x = P - Pi$$
$\therefore Pi - x = 2Pi - P$

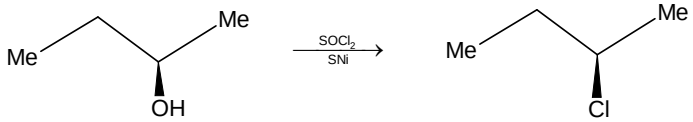
23. B
Sol. Lyophobic sols are easily coagulated

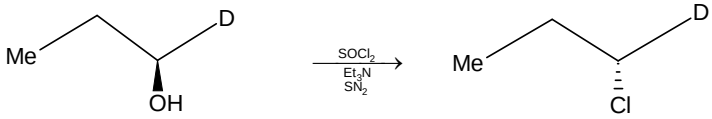
24. A
Sol.

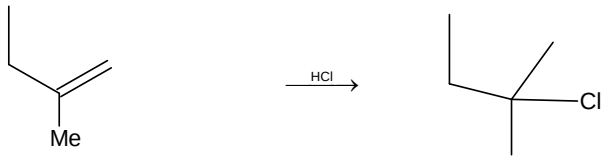


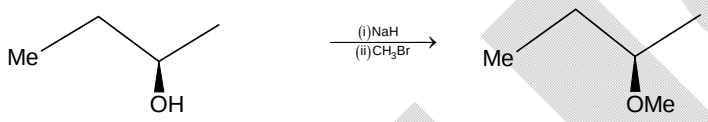
25. A

Sol.









26. B

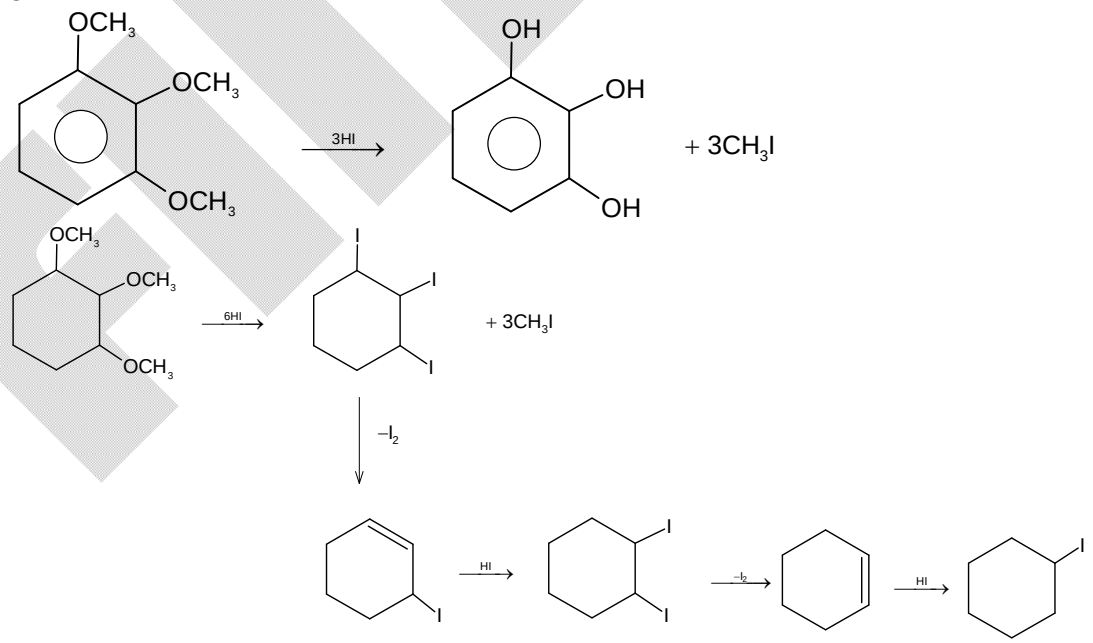
Sol. Refer mechanism of reactions

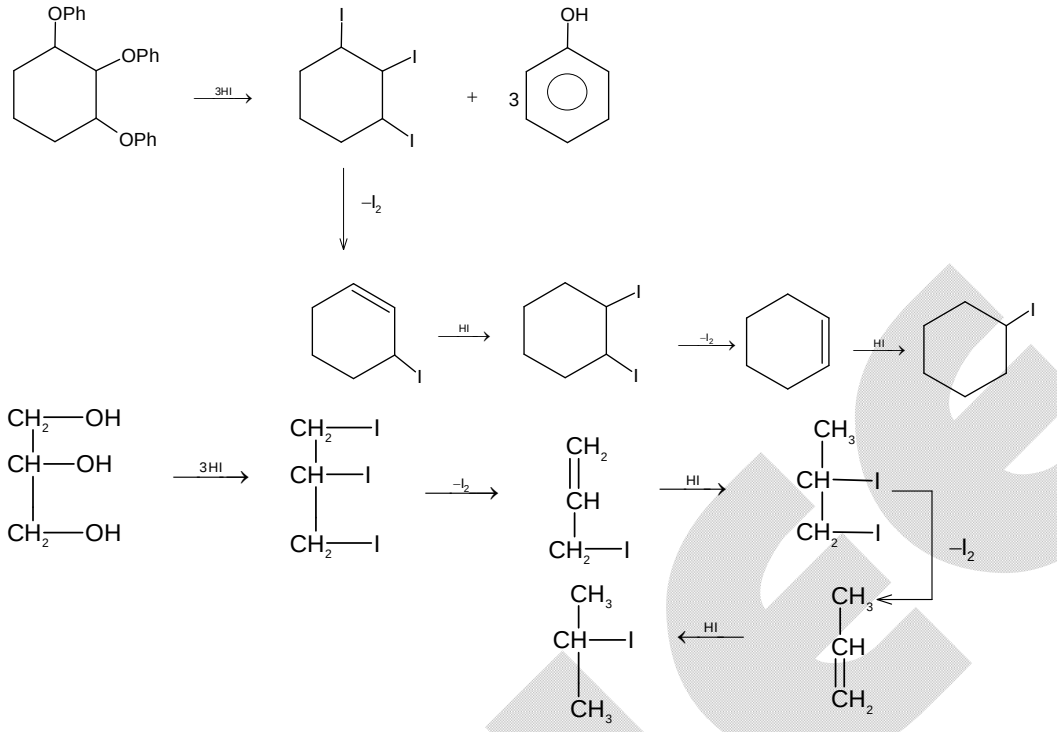
27. A

Sol. Factual

28. C

Sol.





SECTION – B

29. 66

Sol. $\frac{W}{88}\times 2=\frac{1}{20}\times\frac{90}{1000}$

$W = 0.198$

% of oxalate = $\frac{0.198}{0.3}\times 100 = 66\%$

30. 2

Sol. $\frac{\Delta[B]}{\Delta[A]} = n$

$n=\frac{0.6}{0.3}=2$

31. 124

Sol. $CH_4 = xmL$

$C_2H_6 = (100–x)$ mL

$CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(l)$

$-xmL \quad -2xmL \quad +xmL$

$C_2H_6(g) + \frac{7}{2}O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$

$-(100-x) \quad -\frac{7}{2}(100-x) \quad +2(100-x)$

$212=(x+2x-x)+\left[100-x+\frac{7}{2}(100-x)-2(100-x)\right]$

$212 = 2x + 100 - x + \frac{700}{2} - \frac{7}{2}x - 200 + 2x = 250 - \frac{x}{2}$

$\frac{x}{2} = 38 \Rightarrow X = 76\,mL$

$\therefore$ contraction due to kOH = V_{CO2} = x + 2 (100-x) = 200-x= 124 mL

32. 435

Sol. $r_n = 0.529 \times \frac{n^2}{Z}$ Å

$\Rightarrow 10^5 = 0.529\ n^2$

$n^2 = 1.89 \times 10^5 = 18.9 \times 10^4$

$n = 434.7 \approx 435$

33. 5

Sol. $pK_b = 9 \log 1\text{-}5 = 8.824$

$pOH = pKb + \log \frac{C_2}{C_1} = 8.824 + \log\frac{0.15}{0.1} = 9$

pH = 14–9 = 5

34. 392

Sol. $\ln\frac{10}{1} = \frac{2700}{R}\left(\frac{1}{307} - \frac{1}{T}\right)$

$T \approx 392$ K

Mathematics

PART – III

SECTION – A

35. A, D

Sol. Since $\int_a^b f(x)dx = (b-a)\int_0^1 f\{(b-a)x+a\}dx$,

$$\int_1^2 \sin x^2 dx = \int_0^1 \sin(x+1)^2 dx = \int_0^1 \sin(x^2+2x+1)dx$$

$$\int_{-4}^4 \cos x^2\, dx = 8\int_0^1 \cos(8x-4)^2\, dx$$

$$= 8\int_0^1 \cos 16(2x-1)^2 \quad dx.$$

36. A, B, D

Sol. $2^x + 2^{2x+1} = (y-1)(y+1)$

Shows that the factors $y-1$ and $y+1$ are even, exactly one of them divisible by 4. Hence $x \geq 3$ and one of these factors is divisible by 2^{x-1} but not by 2^x.

37. B, C, D

Sol. $$|\ell(z)| = |Az+B| = \frac{1}{2}|(z+1)(A+B)+(z-1)(A-B)| = \frac{1}{2}|(z+1)\ell(1)+(z-1)\ell(-1)|$$

$$\leq \frac{1}{2}(|z+1|+|z-1|)M = \frac{1}{2}km$$

38. C

Sol. $$A = \lim_{x\to 0}\frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$$

$$\lim_{x\to 0^-}\frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)} = \lim_{x\to 0^{-1}}\frac{x}{-x} = -1$$

$$\lim_{x\to 0^+}\frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)} = \lim_{x\to 0}\frac{x}{x} = 1$$

$\Rightarrow$ A does not exist

$$B = \lim_{x\to 0}\frac{[|x|]}{x} = 0 \text{ as } [|x|] = 0 \text{ when } x \to 0.$$

39. A

Sol. In order to check for correct option we need to find cases for $a^2 + b^2 - c^2$ to be positive, negative or zero

Cresting table for

c^2	$a^2 + b^2 < c^2$	(a, b)
1	0	
4	1	(1, 1)
9	3+1	(1, 2) (2, 1), (2, 2)
16	4 + 3 + 1	(1, 3) (3, 1), (2, 3), (3, 2)
25	5 + 4 + 3 + 1	(2, 4) (4, 2) (1, 4) (4, 1) (3, 3)
36	9 + 5 + 4 + 3 + 1	(5, 3) (3, 5) (2, 5) (5, 2) (1, 5) (5, 1) (4, 4) (3, 4) (4, 3)

40. A

Sol. $T_r = \frac{3^r}{r(r+1)} - \frac{3^{r+1}}{(r+1)(r+2)}$

41. A

Sol. $r^2 = \left(x_1 - \frac{y_1}{m_1}\right)^2 \left(\frac{5}{4}\right) = \frac{\left(x_1 + \frac{y_1}{m_1}\right)^2}{4} + \left(x_1 - \frac{y_1}{m_1} - y_1\right)^2$

$\Rightarrow y^2 + \frac{dy}{m} + \frac{2y^2}{m} - 2xy = 0$

$-y^2 + 2xy = \frac{y}{m}(x + 2y)$

$m = \frac{x+2y}{2x-y}$

$\frac{dy}{dx} = \frac{x+2y}{2x-y}$

42. A

Sol. $P \to 3,\ Q \to 2,\ R \to 1,\ S \to 4$

(P) Required probability = $\frac{1}{2}$.

(Q) Required probability $= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^7 = \frac{47}{2^7}$.

(R) Required probability $= \frac{4}{2^7}$.

(S) There are five cases 0, 1, 3 or 4 wins

Required probability $= \frac{1+7+15+10+1}{2^7} = \frac{34}{2^7}$.

43. B

Sol. $P \to 1, 3;\ Q \to 2;\ R \to 4;\ S \to 2$

$I(m) = \int_0^{\pi} \ln(1 - 2m\cos x + m^2)\,dx$

$I(-m) = \int_0^{\pi} \ln(1 + 2m\cos x + m^2)\,dx = \int_0^{\pi} \ln(1 + 2m\cos(\pi - x) + m^2)\,dx$

$= \int_0^{\pi} \ln(1 - 2m\cos x + m^2)\,dx = I(m)$

$\Rightarrow I(m) = I(-m)$ .....(1)

Also $I(m) + I(-m) = \int_0^{\pi} \ln(1 - 2m^2\cos 2x + m^4)\,dx$

Put $2x = t$

$\therefore I(m) + I(-m) = I(m^2)$ ....(2)

$2I(m) = I(m^2) = \frac{I(m^2)}{I(m)} = 2.$

44. D

Sol. Function is even with $f'(x)=\frac{dy}{dx}=\tan\frac{\theta}{2}$. Hence f(x) is not differentiable at x = $\pm\pi$, $\pm3\pi$.....(k = π).

Also $f(x+2a\pi)=f(x)$ same for f'(x)

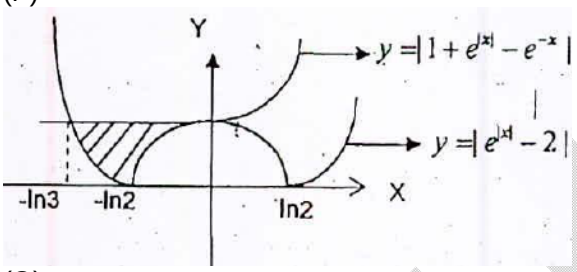
Area $=2\int_0^{2a} xdy=2\int_0^{\pi} a(\theta+\sin\theta)(a\sin\theta)=3\pi a^2$

$x=a(\theta+\sin\theta),\ y=a(1-\cos\theta);\ \frac{dx}{d\theta}=a(1+\cos\theta)\Rightarrow\frac{dy}{d\theta}=a\sin\theta$

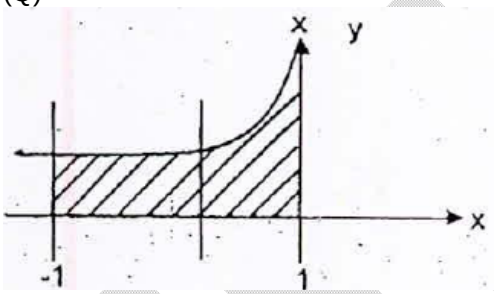
$\frac{dy}{dx}=\frac{\sin\theta}{1+\cos\theta}=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}=\tan\frac{\theta}{2}$ not differentiable at $\theta=\pi$ and $\theta=-\pi$

45. A

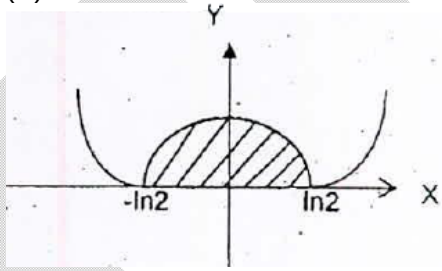
Sol. (P)



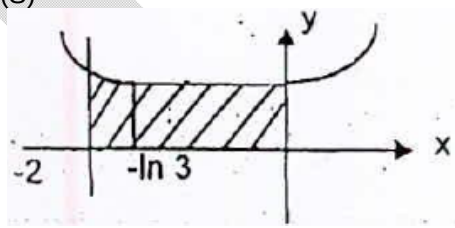
(Q)



(R)



(S)



SECTION – B

46. 2

Sol. With respect to each x, y, z coordinate picked in two distinct points the parity of numbers should be same. No. of such triplets = $(2^2+1^2)(2^2+2^2)(3^2+2^2)-3\times4\times5$

Probability = $\frac{460}{3540}=\frac{23}{177}$

47. 5

Sol. $b\cos x+\frac{b\cos 3x}{2}=b\cos x\left(2\cos^2 x-\frac{1}{2}\right)$

$=|b\cos x|\left|\frac{3}{2}-2\sin^2 x\right|$

$\le\frac{3}{2}|b|\le 3$

$\Rightarrow |b|\le 2 \Rightarrow b=-2,-1,0,1,2.$

48. 1

Sol. $\frac{2f(x).f'(x)}{\sqrt{1-(f(x))^4}}-2x\ge 0\Rightarrow\frac{d}{dx}(\sin^{-1}(f(x))^2-x^2)\ge 0$

Let $g(x)=\sin^{-1}((f(x))^2)-x^2$ is a non-decreasing function.

$\Rightarrow \lim_{x\to x_1} g(x)\le\lim_{x\to x_2} g(x)\Rightarrow\frac{\pi}{2}-x_1^2\le\frac{\pi}{6}-x_2^2$

$\Rightarrow x_1^2-x_2^2\ge\frac{\pi}{3}\Rightarrow\left[x_1^2-x_2^2\right]\ge 1.$

49. 8

Sol. Above expression is $\sum\left(2x^2+(1-x)^2\right),\ x\in[0,1]$

$x\in(p, q, r, s)$

Consider $f(x)=2x^2+(1-x)^2$

50. 1

Sol. If the sum of digits in a number is divisible by 9, then number itself is divisible by 9

$\Rightarrow$ either number comprises {1, 2, 4, 5, 6} or {1, 2, 3, 5, 7}.

$\Rightarrow$ required probability = $\frac{2\times5!}{{}^7P_5}=\frac{2}{21}\Rightarrow\frac{x}{y}=\frac{2}{21}\Rightarrow y-10x=1.$

51. 2

Sol. $T_n=\cot^{-1}\left[2^{n+1}+\frac{1}{2^n}\right]\Rightarrow T_n=\cot^{-1}\left[\frac{2^{2n+1}+1}{2^n}\right]$

$T_n=\tan^{-1}\left[\frac{2^n}{1+2^{2n+1}}\right]\Rightarrow T_n=\tan^{-1}\left[\frac{2^{2n+1}-2^n}{1+2^{n+1}.2^n}\right]$

$T_n=\tan^{-1}2^{n+1}-\tan^{-1}2^n$

$S_n=\sum_{n=1}^{n}T_n$

$S_n=\tan^{-1}(2^{n+1})-\tan^{-1}(2)$

$S_n=\frac{\pi}{2}-\tan^{-1}2=\cot^{-1}2$